Semi-analytic approach to diverted tokamak equilibria with incompressible toroidal and poloidal flows

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Abstract
Generalized Grad–Shafranov equation for tokamak equilibrium with incompressible toroidal and poloidal flows is solved to obtain a double-null diverted configuration based on an approach presented before (Shi 2008 Plasma Phys. Control. Fusion 50 085006). This solution consists of only two terms of base functions obtained from the variable-separating method and suitable for describing both the internal region within the separatrix and a scrape-off layer region. Dependences of the main equilibrium properties, such as the magnetic field, plasma pressure and the equilibrium radial electric field and the plasma current on flows are revealed. In particular, we find that the presence of poloidal flow causes a deviation of the current surface from the magnetic surface and the sheared poloidal flow produces a non-zero toroidal current component that possibly affects the peeling–ballooning stability of the pedestal.

1. Introduction
Equilibrium properties of tokamak plasmas with both toroidal and poloidal flows have attracted considerable interest continuously [1–17]. Both toroidal flows and poloidal flows are observed in many conventional tokamaks [18,19] and in small aspect ratio tokamaks (STs) [20–22]. Shears of these flows are thought to be important for the stability of tokamak plasmas and related to the mode transition from the low confinement (L-mode) to the high confinement (H-mode). One of the purposes of calculating the stationary equilibria is to investigate their magnetohydrodynamic stability. It is found that new Alfvén continuum gaps and global modes can be induced by the toroidal flow [12,13] and there are unstable continuous spectra due to poloidal flows exceeding the critical slow magnetosonic speed [14], and the corresponding spectral code is developed [15]. Previous theoretical investigations have drawn several tentative conclusions: (i) equilibria with pure toroidal or parallel flows are relatively easier to treat by a simple transformation of the poloidal magnetic flux to a generalized flux function that is basically governed by an equation similar to the original Grad–Shafranov equation for the no-flow case, and the main physical results are that the contours of the scalar pressure depart from the magnetic surfaces [11]). (ii) For poloidal flows, it is shown that equilibrium with a pure poloidal flow is not permitted at least in the single MHD case. (iii) For both toroidal and poloidal flows, if the poloidal flow is supersonic, there are discontinuities in the Bernoulli equation and generally, equations involved could be of elliptic or hyperbolic type according to some criteria. These equations are usually solved numerically (e.g. stationary equilibrium solver FINESSE [16]) and the analytic solution is so far only for the circular cross-section case (at the lowest order of expansion) by the inverse aspect ratio expansion techniques [14,17,22]. For incompressible flows, the Bernoulli equation is simplified and we can avoid solving the hyperbolic-type partial differential equation at least for tokamak equilibria with both toroidal and poloidal flows (generally in the whole plasma region the governing partial differential equation is elliptic). Incompressibility makes an analytic or semi-analytic approach to equilibrium equations with both toroidal and poloidal flows feasible, because in this case, we can use a transformation to obtain an extended poloidal flux function that satisfies an extended Grad–Shafranov equation as indicated in [4]. Starting from this equation, to suitably specify three surface functions, the generally non-linear partial differential equilibrium equation can be turned into a linear one. Then we use a novel methodology developed recently [23,24] to solve this equation to obtain a diverted tokamak-like configuration.

This paper is arranged as follows. In section 2, basic equations are presented. In section 3 we use the