3.8 Tokamak residual zonal flow level in near separatrix region

SHI Bingren

The \( E \times B \) zonal flows are generally recognized to be significant for the reduction of turbulent transport either in tokamaks or in other toroidal plasma confinement systems. This is an issue closely relevant to the confinement mode transition from the low (L-mode) to the high (H-mode), and the standard transition usually appears in the near edge region. For most of tokamak devices, such transition happens in an equilibrium with divertor configuration. Previously, Rosenbluth and Hinton provide an analytic study of the ITG turbulence for tokamak plasmas and obtain an analytic formula for the residual zonal flow that is suitable for large aspect ratio circular tokamak plasma. Later, non-circular shaping and arbitrary aspect ratio effects are considered by using a simple analytic equilibrium model, the so-called Solov’ev configuration. However, the more experimentally relevant case, the residual zonal flow level in the divertor configuration is still not considered. Very recently, we suggest to use a semi-analytic simple model to describe the tokamak configuration with divertor separatrix that lends us convenience to calculate this residual zonal flow level in the near separatrix region.

Tokamak equilibrium of up-down symmetry with D-shape divertor separatrix can be described by a semi-analytic model. In this model, the poloidal flux \( \psi (R,Z) \) can be simply described by only two terms of separable solutions of the original Grad-Shafranov equation:

\[
\psi(R,Z) \approx \psi_1 (x) \cos ky + \psi_2 (x) \sin ky
\]

The poloidal magnetic field is determined by

\[
B_p = \nabla \psi \times \nabla \zeta
\]

And \((R,\zeta, Z)\) is the cylindrical coordinates while \(x = R/R_0\) and \(y = Z/R_0\) with \(R_0\) being the major radius of the equilibrium configuration. Function \(g_k(x)\) is the solution of the following homogenous ordinary differential equation:

\[
\frac{d^2 g_k}{dx^2} - \frac{1}{x} \frac{dg_k}{dx} + \left( \frac{\eta}{x} + \frac{\eta}{x} \right) g_k = 0
\]

While \(g_0(x)\) is the solution of the following equation

\[
\frac{d^2 g_0}{dx^2} + \frac{1}{x} \frac{dg_0}{dx} + \left( \frac{\eta}{x} + \gamma k^2 \right) g_0 = 0
\]

Eqs. (3) and (4) will be solved numerically in range \(0 < x_1 \leq x \leq x_2\) (For the details please refer to ). In this paper, for simplicity we select \(x_1 = x_{i2} = x_i\), so that the vertical line \(x = x_i\) will be automatically a sector of the boundary magnetic surface \(\psi = 0\) that exhibits a D-shape structure. There is a null point at this boundary magnetic surface for the poloidal magnetic field where

\[
\frac{\partial \psi}{\partial x} = g_k'(x_i) \cos ky + c_0 g_0'(x_i) = 0
\]

This condition means

\[
y_\phi = \frac{1}{k} \cos^{-1} \left[ -\frac{c_0 g_0'(x_i)}{g_k'(x_i)} \right]
\]

With the Increase of the coefficient \(c_0\) from \(c_0 = 0\) to a critical value \(c_0 = \frac{g_k'(x_i)}{g_0'(x_i)}\), the X-point moves away from \(y_\phi = \pi/2k\) to \(y_\phi = \pi/k\) and the elongation of the last closed magnetic surface (defined as the ratio of the area enclosed by this surface divided by the area of a circle that has a radius \((x_2 - x_1)/2\) and \(x_2\) is the outermost major radius of the boundary surface) also increases gradually.

By setting \(x_i = 0.65\) we obtain tokamak equilibrium with \(A = 3.2\) and the allowable largest elongation \(\kappa = 2.5\) for \(k = 4\) and \(c_0 = 0.696\). The configuration in this way is shown in Fig. 1.

![Magnetic surface for equilibrium with parameters indicated](image)

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Recently, for equilibrium of arbitrary aspect ratio and large elongation, we find that the residual zonal flow level can be expressed by a formula quite similar with the original Rosenbluth-Hinton’s if we define the safety factor \( q \) as the cylindrical one for circular cross-section case but with arbitrary aspect ratio. In this paper, we carry out a similar calculation for equilibrium with double divertor structure described by semi-analytic solution. The starting equation is the same:

\[
\varphi_{\text{L}} = \varphi_{\text{in}} / \Theta
\]  

(7)

Where \( \varphi_{\text{in}} \) is the Fourier component of the electric potential of the zonal flow, the subscript “f” and “0” represent the final state and the original state, respectively, \( \Theta \) is the shield factor due to the neoclassical polarization of ions:

\[
\Theta = 1 + \frac{(FS')^2}{k x^2} < \langle \psi^2 \rangle > \left[ < \langle \psi^2 \rangle > - \frac{3}{2} \int A g(\lambda) d\lambda \right]
\]  

(8)

Where \( h = B_r / R = x \), for configuration of interest in this paper, i.e., that described in previous section, we have

\[
< \langle \psi^2 \rangle > = \frac{1}{\eta} \int \frac{x^2 \sqrt{r' + r_0^2} d\theta}{\sqrt{(g_\psi \cos k y + c_0 g_{\psi}^0)^2 + h^2 \sin^2 k y g_\psi^0}} / A
\]  

(9)

\[
A = \frac{1}{\eta} \int \frac{(g_\psi \cos k y + c_0 g_{\psi}^0)^2 + h^2 \sin^2 k y g_\psi^0} {\sqrt{(g_\psi \cos k y + c_0 g_{\psi}^0)^2 + k^2 \sin^2 k y g_\psi^0}} d\theta / A
\]  

(10)

\[
< \langle \psi^2 \rangle > = \frac{S^2}{\eta} \int \frac{x^2 \left[ (g_\psi \cos k y + c_0 g_{\psi}^0)^2 + k^2 \sin^2 k y g_\psi^0 \right]} {\sqrt{(g_\psi \cos k y + c_0 g_{\psi}^0)^2 + k^2 \sin^2 k y g_\psi^0}} d\theta / A
\]  

(11)

The function

\[
g(\lambda) = 1/ \int \frac{d\theta}{\sqrt{h(\lambda - A) / \eta}} , \quad 0 \leq A < A_L = x_L
\]  

(12)

is related to the contribution of the trapped particles.

Near the magnetic axis, we use approximate analytic magnetic surface that exhibits elliptic cross-section and satisfies the following equation:

\[
\frac{(x - x_0)^2}{a^2} + \frac{x^2}{b^2} = 1
\]

Or

\[
x = x_0 + a \cos \gamma, \quad y = b \sin \gamma
\]  

(13)

with

\[
a = \sqrt{ \frac{2(\psi_{in} - \psi)}{(g_\psi' + c_0 g_{\psi}^0)^2 + k^2 g_\psi^0}} \quad b = \frac{2(\psi_{in} - \psi)}{k^2 g_\psi^0}
\]  

(14)

And along the poloidal projection of the magnetic field line we have

\[
dl / B_p = x dx / ( - \partial \psi / \partial y )
\]

\[
= (xa \sin t / k^2 g_\psi(x) b \sin t) dt
\]

(15)

This approximation facilitates calculations of magnetic relevant integrals.

Finally we obtain

\[
\Theta = 1 + \frac{q_0^2}{\rho_0^2} \int < \langle \psi^2 \rangle > \left[ < \langle \psi^2 \rangle > - \frac{3}{2} \int A g(\lambda) d\lambda \right]
\]

(16)

The increase in the shielding factor \( \Theta \) (so that the decrease of the residual zonal flow level) when \( \epsilon \) decrease from about a value \( \epsilon = 0 \) is generally observed in other similar analyses and is the general result of neoclassical polarization.

Unlike the analytic configuration such as the Solov’ev solution that permits a separation of the elongation from other parameters, now we can hardly do so. For equilibrium cited above, change of elongation would result in change of other parameters like the triangularity. A clear trend is seen, however, that near a separatrix, the neoclassical shielding effect goes strong and the residual zonal flow level goes weak. This conclusion is to somewhat contradicts to the general observation that zonal flows stabilize turbulence and make transition from the L-mode to the H-mode and a concomitant large decrease in the edge electric field to a relatively large negative one. Then, we think, more physics should be sought besides what had been discussed in recent literatures regarding the zonal flows excited by ITG turbulence.