3.3 Effects of turbulence induced viscosity and plasma flow on resistive wall mode stability

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The stability of the resistive wall mode (RWM) is of great importance for steady state advanced tokamaks. The stabilization physics of flow on RWM has not been completely resolved, partly because the recent experiments show that the existing magnetohydrodynamic (MHD) theory on RWM is insufficient consummate. In this paper, the eigenmode equation for RWM is derived on the basis of previous work to further understand the stabilization physics of flow on RWM. In addition to the plasma flow $V$ and the parallel viscosity $\eta_p$, a new dissipation mechanism which has not been considered in the RWM physics of tokamak plasmas, turbulent viscosity $\chi$, is taken into account in the equation. The present numerical results indicate that the turbulent viscosity $\chi$ reduces significantly the threshold flow velocity required for RWM stabilization.

A cylindrical plasma with the minor radius $r = a$ and the major radius $r = R$ is considered, surrounded by a resistive wall at the minor radius $r = b$ with the wall thickness $d$ and conductivity $\sigma$. The plasma displacement is expressed as

$$\xi = \xi_1(r) \exp(-i\omega t + ikz + im\theta)$$  \hspace{1cm} (1)

We extend the previous RWM theory for reversed field pinches to tokamaks. Starting with the linearized MHD equations and carrying out the algebraic operation as that in, we obtained the following eigenmode equation for the RWM:

$$C_2 \frac{\partial^2 \psi}{\partial r^2} + C_1 \frac{\partial \phi}{\partial r} + C_0 \phi = 0$$  \hspace{1cm} (2)

Where $\psi = r \xi_1$, $C_0$, $C_1$, and $C_2$ are the functions of $r$, $\omega$, $\chi$, $\Omega_b$, $\eta_p$, $k$, $m$, and $\rho_0$. Here, $\omega = \omega_0 + i\gamma$ is the eigenfrequency, $\Omega_b = k \cdot V$ is the plasma toroidal rotation frequency, $\rho_0$ is the plasma density. Here the frequencies $\omega$ and $\Omega_b$, the coefficients $\chi$, $\eta_p$, $\sigma$, the length scales $b$, $d$, and $1/k$ are normalized to $\omega_\lambda$, $\alpha V_\lambda$, $\rho_0 a V_\lambda$, $1/\omega_\lambda \rho_0 a^2$, and $a$, respectively, where $\omega_\lambda = \frac{V_\lambda}{R}$ and $V_\lambda = \frac{B_0}{\sqrt{\rho_0 \mu_0 \rho_0}}$

We solve numerically the eigenvalue problem with boundary conditions given in the following. At the plasma-vacuum interface, the perturbed radial magnetic field and the perturbed pressure are continuous. At the wall position, the perturbed magnetic field $B'_r$ in the vacuum is continuous across the wall surface, with $B'_r$ satisfying the thin wall jump condition. As a test case, we consider a large aspect ratio equilibrium configuration, with $B_\phi =$ constant, $B_\parallel = B_\parallel(1 - (1 - r^2)^2)/r$, $J = [0, 0, J_b (1 - r^2)]$, $\nabla P = J \times B$ and $R/a = 10$.

Firstly, we carry out the test computations in the case without the parallel viscosity, the turbulent viscosity and plasma flow. Calculations show that, at high enough wall conductivity, the mode growth rate scales inversely proportional to the conductivity. We also find a critical wall radius $b_c$, above which the RWM is unstable even in the presence of a perfectly conducting wall. These results from test computations are in agreement with the previous theories.

We then study the effects of turbulent viscosity $\chi$ on the RWM stability without the plasma flow. The coefficient $\chi$ is taken to be a constant along the minor radius. The value of $\chi$ in this study is estimated based on the experimental measurements. The numerical results indicate that the growth rate of the RWM decreases quickly with increase of $\chi$.

![Fig. 1. The critical toroidal rotation frequency versus the turbulent viscosity coefficient.](image)

The effects of the $\chi$ on the RWM growth rate are also

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investigated in the presence of the plasma flow. It is assumed that the plasma flow $V$ is only in the $Z$ direction and is uniform along the minor radius. The numerical results show that the mode is completely suppressed as the plasma rotation frequency exceeds a critical value $\Omega_c$. In addition, it is found that, when the mode starts to be stabilized, the real frequency of the mode, $\omega_r$, tends to saturate and finally decays. Calculations also exhibit that the mode frequency is proportional to the viscosities.

The critical rotation frequency $\Omega_c$ significantly decreases with increase of the $\chi$, as shown in Fig. 1. The other parameters assumed in Fig. 1 are $q_s = 1.6$, $\sigma = 10^7$, $b = 1.1$, $d = 0.01$, $m = 2$ and $k = -0.1$. Furthermore, Fig. 1 indicates that the increase of parallel viscosity also reduces the critical toroidal rotation frequency.

The effects of the turbulent viscosity and the plasma rotation frequency on the stability window have also been investigated. For a given $\eta_t$ and a proper choice of $\Omega_c$, we calculate the growth rate of the mode varying the wall distance $b$ for various $\chi$. It is found that, when the turbulent viscosity reaches a certain value, the stability window first appears at an intermediate location between the plasma edge and the critical wall distance ($b_c$) for the ideal kink mode with ideal wall. A further increase in $\chi$ widens the stability window toward the plasma boundary. These computations show that the turbulent viscosity have significant influence on the stability window in the terms of the wall minor radius.

To gain more complete understanding of the RWM stability, it would be useful to extend the present model to a more realistic situation, such as including Alfvén/sound continuum damping, drift kinetic effects, the shear flow effect, which are planned to be investigated using MARS-F($K$) code in the future.